QusCos[1]

Quantized State System Simulation in Scicos

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3rd International Workshop on Simulation at the System Level for Industrial Applications

October 14th to 16th, 2015

ENS Cachan (France)

[1] speak 'Couscous'

Outline

Introduction

- In 2001, Quantized State Systems were introduced
  - accompagnied by a first order integration method (QSS1)
- Today, there are different integration methods (solvers) available:
  - QSS (order 1...4; non-stiff problems)
  - CQSS (order 1 only; marginally stable problems)
  - LIQSS (order 1...4; stiff problems)
- Traditionally, implementations are based on the DEVS formalism
  - we may regard PowerDEVS as a reference implementation
- In general, QSS methods
  - have pleasing convergence, stability and error bound properties
  - update system states asynchronously
  - need no iteration scheme for discontinuity handling
  - require explicit calculations only (for ODE solving w/ method order $\leq$4)
- Focus of this talk:
  - proof-of-concept implementation of QSS1 & QSS2 methods in Scicos
  - illustration of QSS peculiarities (no mathematical subtleties)

Quantized State System (QSS)

A continous time invariant system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \tag{1}$$

with $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ is approximated by a QSS

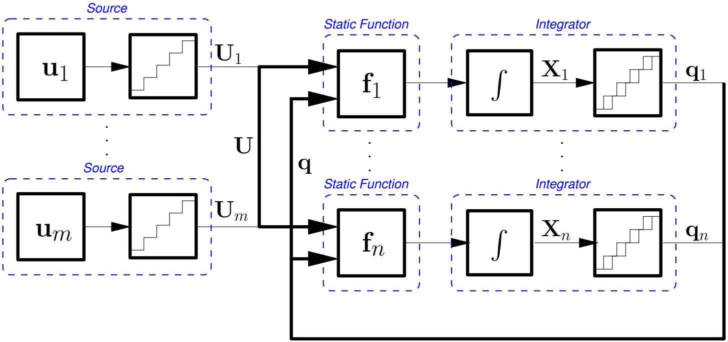
$$\dot{\mathbf{X}}(t) = \mathbf{f}(\mathbf{q}(t), \mathbf{U}(t)) \tag{2}$$

with[2] $\mathbf{X}, \mathbf{q} \in \mathbb{R}^n$ and $\mathbf{U} \in \mathbb{R}^m$, where

- $\mathbf{x}(t)$ is replaced by the quantized state $\mathbf{q}(t)$
- $\mathbf{U}(t)$ is a quantized approximation of $\mathbf{u}(t)$
- $\dot{\mathbf{X}}(t)$ is an approximation of $\dot{\mathbf{x}}(t)$
- $\mathbf{q}(t)$ and $\mathbf{X}(t)$ are related by hysteretic quantization
- changes of quantization levels are events
- eq. (2) specifies a legitimate discrete event system

[2] In literature, usually the same notation is used for $\mathbf{x}$ and $\mathbf{X}$ or $\mathbf{u}$ and $\mathbf{U}$

Block Diagram



- Source, Static Function and Integrator are the building blocks
- For simplicity, focus on blocks with scalar inputs/outputs only
- How does/could an Integrator block work?

First Order Integration Method (QSS1, Scalar)

Choice of hysteretic quantization for $X, q \in \mathbb{R}$ as

$$q(t) = \begin{cases} X(t) & \text{if } |q(t^-) - X(t)| = \Delta Q, \\ q(t^-) & \text{otherwise} \end{cases} \tag{3}$$

with the quantum $\Delta Q > 0$. From eq. (3) follows:

- $q(t)$ is piecewise constant
- $q(t)$ changes only, when it differs from $X(t)$ by ΔQ

Quantized input $U(t) \in \mathbb{R}$ is piecewise constant. From eq. (2)[3] follows:

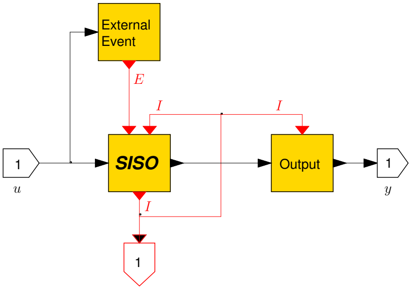
- $\dot{X}(t)$ is piecewise constant,
- thus integration is easy, and $X(t)$ is piecewise linear

Second order QSS2 method works likewise[4].

[3] $\dot{X}(t) = f(q(t), U(t))$

[4] with piecewise linear $q(t)$, $U(t)$, $\dot{X}(t)$, and piecewise parabolic $X(t)$

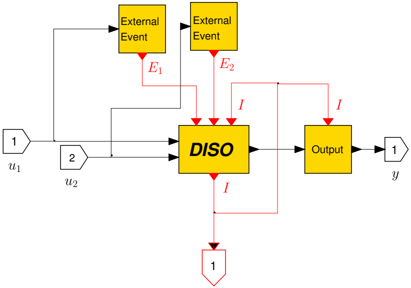
Anatomy of Blocks (SISO)



Single Input / Single Output:

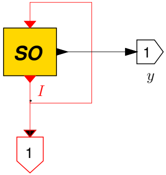
- e.g. Integrator, Gain, . . .
- **External Event** is generic
- **Output** is generic
  - no state
  - direct feedthrough
  - purpose: event filtering
- **SISO** is operation-specific
  - discrete state
  - no direct feedthrough
  - ***E*** or ***I*** triggers state update
  - only ***I*** triggers output update
- ***E*** and ***I*** are asynchronous

Anatomy of Blocks (DISO, SO)



Dual Input / Single Output:

- e.g. Sum, Product, . . .
- ***E*$_1$** and ***E*$_2$** are not synchronized with ***I***
- ***E*$_1$** and ***E*$_2$** might be synchronous
- scheme allows up to 31 inputs



Single Output (Source):

- e.g. Step, Pulse, . . .
- **Output** block is not needed

Representation of Quantities

QSS method of order $k = 1, 2, \ldots$:

- input/output signal **v** of a block

$$\mathbf{v} \in \mathbb{R}^k$$

- approximated state **X** used inside an integrator block

$$\mathbf{X} \in \mathbb{R}^{k+1}$$

- components of a quantity $\mathbf{a} = (a_1, a_2, a_3, \ldots)$
    - a_1 : value
    - a_2 : slope (1st derivative)
    - a_3 : curvature (2nd derivative)
    - $\ldots$

Static Function (Product, QSS2)

Inputs $\mathbf{u_1}, \mathbf{u_2} \in \mathbb{R}^2$, Output $\mathbf{y} \in \mathbb{R}^2$
State $(\mathbf{v_1}, \mathbf{v_2}, t_l, \sigma) \in \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R}$

Time of Last State Update t_l, Lifespan of State σ

Initialization

$$\mathbf{y} \leftarrow \mathbf{0}$$
$$\mathbf{v_1} \leftarrow \mathbf{u_1}$$
$$\mathbf{v_2} \leftarrow \mathbf{u_2}$$
$$t_l \leftarrow 0$$
$$\sigma \leftarrow 0$$

Output Update

$$\boldsymbol{I}: \quad y_1 \leftarrow v_{11} \cdot v_{21}$$
$$y_2 \leftarrow v_{11} \cdot v_{22} + v_{12} \cdot v_{21}$$

State Update & Schedule of ***I***

$$e \leftarrow t - t_l$$
$$t_l \leftarrow t$$
$$\boldsymbol{E_1} \wedge \overline{\boldsymbol{E_2}}: \quad \mathbf{v_1} \leftarrow \mathbf{u_1}$$
$$v_{21} \leftarrow v_{21} + v_{22} \cdot e$$
$$\overline{\boldsymbol{E_1}} \wedge \boldsymbol{E_2}: \quad \mathbf{v_2} \leftarrow \mathbf{u_2}$$
$$v_{11} \leftarrow v_{11} + v_{12} \cdot e$$
$$\boldsymbol{E_1} \wedge \boldsymbol{E_2}: \quad \mathbf{v_1} \leftarrow \mathbf{u_1}$$
$$\mathbf{v_2} \leftarrow \mathbf{u_2}$$
$$\boldsymbol{E_1} \vee \boldsymbol{E_2}: \quad \sigma \leftarrow 0$$
$$\boldsymbol{I}: \quad \sigma \leftarrow \infty$$
$$\text{next } \boldsymbol{I} \text{ at}: \quad t + \sigma$$

Integrator (QSS1)

Inputs $u \in \mathbb{R}$, Output $y \in \mathbb{R}$
Parameters $(x_0, \Delta Q) \in \mathbb{R} \times \mathbb{R}_+$
State $(\mathbf{X}, q, t_l, \sigma) \in \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

Time of Last State Update t_l, Lifespan of State σ

Initialization

$$\begin{aligned} y &\leftarrow 0 \\ X_1 &\leftarrow x_0 \\ X_2 &\leftarrow 0 \\ q &\leftarrow x_0 \\ t_l &\leftarrow 0 \\ \sigma &\leftarrow 0 \end{aligned}$$

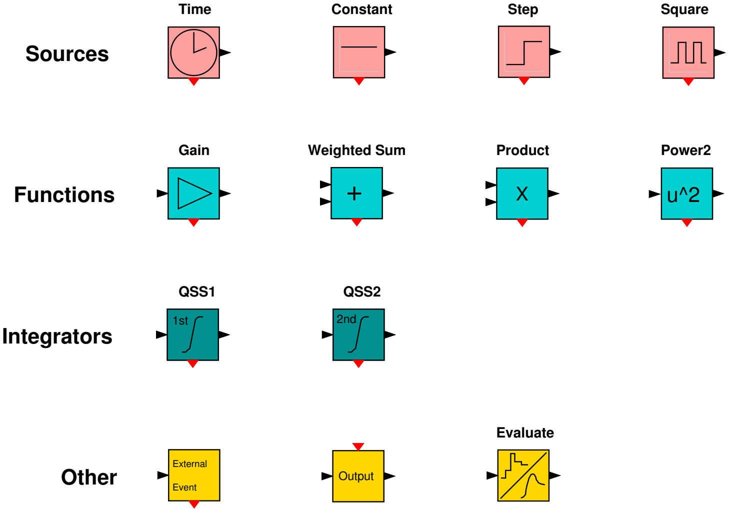
Output Update

$$\boldsymbol{I}: \quad y \leftarrow X_1 + X_2 \cdot \sigma$$

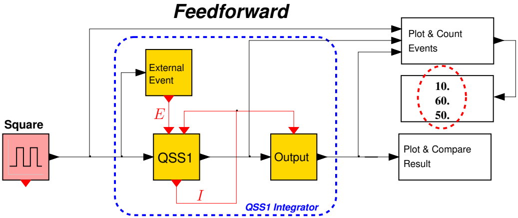
*State Update & Schedule of **I***

$$\begin{aligned} & e \leftarrow t - t_l \\ & t_l \leftarrow t \\ \boldsymbol{I}: \quad & X_1 \leftarrow X_1 + X_2 \cdot \sigma \\ & q \leftarrow X_1 \\ & \sigma \leftarrow \begin{cases} \left|\frac{\Delta Q}{X_2}\right| & \text{if } X_2 \neq 0, \\ \infty & \text{otherwise} \end{cases} \\ \boldsymbol{E}: \quad & X_1 \leftarrow X_1 + X_2 \cdot e \\ & X_2 \leftarrow u \\ & \sigma \leftarrow \\ & \quad \min_{\sigma' \geq 0} \{\sigma' | \Delta Q = |q - X_1 - X_2 \cdot \sigma'|\} \\ \text{next } \boldsymbol{I} \text{ at}: \quad & t + \sigma \end{aligned}$$

Implemented Blocks (QSS1 & QSS2)

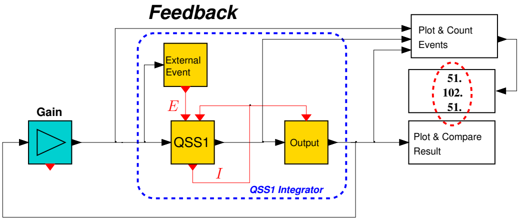


Event Propagation & Filtering (Feedforward)



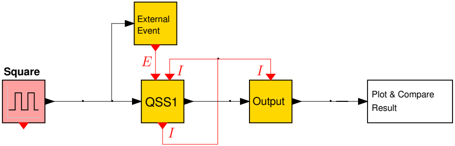
- **Square**
  - generates ***E*** (10)
- **QSS1**
  - generates ***I*** (50)
  - propagates ***E*** and ***I*** (60)
- **Output**
  - filters out ***E*** (10)
  - propagates only ***I*** (50)

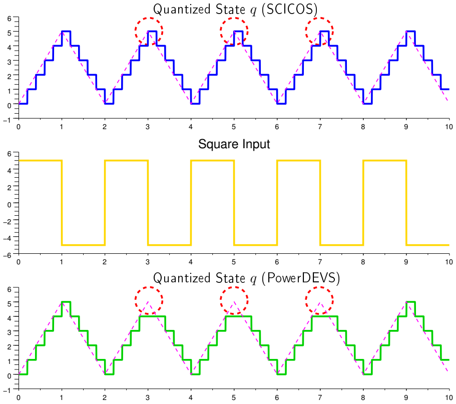
Event Propagation & Filtering (Feedback)



- **Gain**
  - receives ***I*** (51)
  - generates wo/ delay ***E*** (51)
  - propagates only ***E*** (51)
- **QSS1**
  - generates ***I*** (51)
  - propagates ***E*** and ***I*** (102)
- **Output**
  - filters out ***E*** (51)
  - propagates only ***I*** (51)

Indeterminism Effects



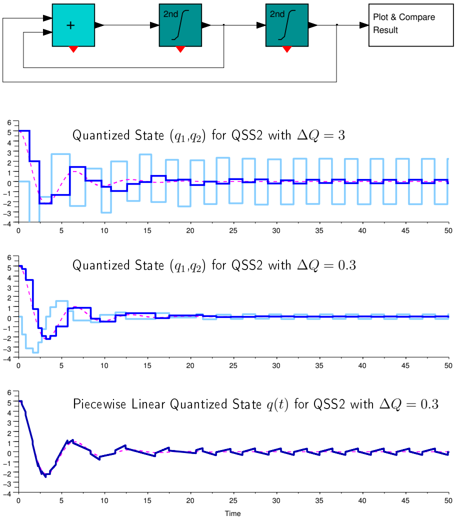


- Example (QSS1 with $\Delta Q = 1$):

  $$\dot{x}(t) = 5\,\mathrm{sign}(\sin(\pi t)),\ x(0) = 0$$

- SCICOS vs. POWERDEVS
  - results are differing
- Why?
- ***E*** and ***I*** at $t = 1, 2, 3, \ldots$
  - simultaneous but asynchronous
  - indetermined processing order
- both results are valid
- different implementations may yield not the very same results

Stability and Ultimately Bounded Oscillations



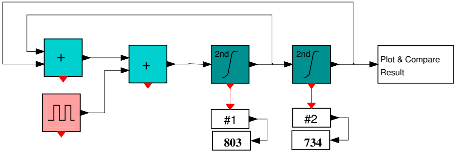
- Example: Stable LTI system

$$\ddot{x}(t)+\frac{1}{2}\dot{x}(t)+x(t) = 0, x(0) = 5, \dot{x}(0) = 0$$

- QSS methods conserve (asymptotic) stability of original system for
  - arbitrary $\Delta Q > 0$ (LTI)
  - sufficient small $\Delta Q > 0$ (nonlinear)
- generated trajectories are
  - practically stable, i.e.
  - ultimately bounded oscillations

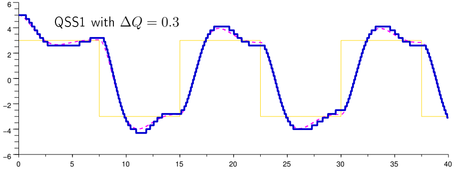
  usually remain at equilibrium points
- calculation of global error bound is
  - easy for LTI systems (explicit formula)
  - possible for nonlinear systems (perturbation theory)

Forced 2nd Order System



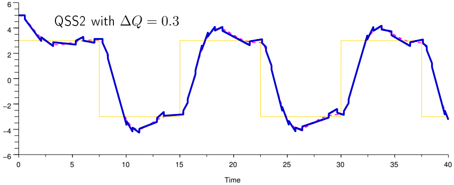
$$\ddot{x}(t) + \dot{x}(t) + x(t) = \operatorname{sign}\left(\sin\left(\frac{2\pi}{15}t\right)\right),$$

$$x(0) = 5,\ \dot{x}(0) = 0$$



State Updates with QSS1 (PowerDEVS)

ΔQ	Integrator #1	Integrator #2
0.1	392 (+1)	412
0.01	3770 (+1)	4066
0.001	37577 (+1)	40609

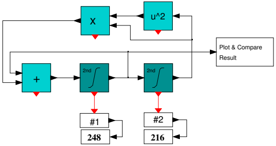


State Updates with QSS2 (PowerDEVS)

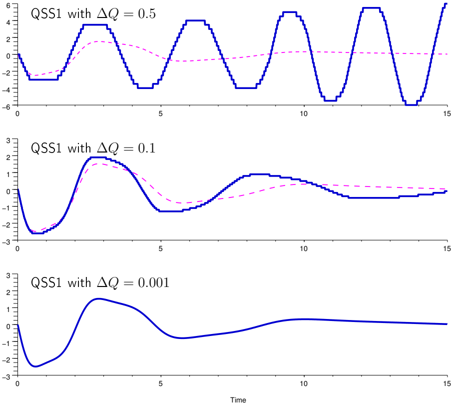
ΔQ	Integrator #1	Integrator #2
0.1	79 (+1)	68
0.01	252 (+1)	229
0.001	803 (+1)	734

(Note: difference (+1) seems to be due to a counting bug)

Nonlinear 2nd Order System (DUFFING Oscillator)



$$\ddot{x}(t) + \frac{1}{3}\dot{x}(t) + x^3(t) = 0,$$
$$x(0) = 2,\ \dot{x}(0) = 0$$



State Updates with QSS1 (PowerDEVS)

ΔQ	Integrator #1	Integrator #2
0.1	143 (+1)	129
0.01	1056 (+1)	965
0.001	10242 (+1)	9431

State Updates with QSS2 (PowerDEVS)

ΔQ	Integrator #1	Integrator #2
0.1	20 (+1)	12
0.01	81 (+1)	64
0.001	248 (+1)	216

(Note: difference (+1) seems to be due to a counting bug)

Conclusions and Outlook

Conclusions:

- implementing QSS methods in SCICOS is feasible
- simulation results are in conformance with POWERDEVS
- starting point for full-featured implementation of QSS methods

Outlook:

- clean-up, optimization, benchmarking with respect to runtime
- possible tweaks of SCICOS for more simulation efficiency
- implementation of
  - illegitimacy detection
  - logarithmic quantization (absolute & relative error control)
  - methods for stiff and marginally stable systems
  - higher order methods
  - switches, hysteresis, saturation, . . .
  - generic static function block
  - vectorial extension of the approach
  - . . .

Merci de votre attention! Questions?

References

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